Indian Statistical Institute, Bangalore

B. Math. Second Year

Second Semester - Graph Theory

Back Paper Exam | Duration: 3 hours | Date : July 14, 2015

Max Marks: 100

1. If G a vertex minimal imperfect graph then show that $\chi(G) = w(G)+1$ and $\chi(G\backslash x) = w(G)$ for all x in $V(G)$. [15]

2. If G_1, G_2 are two graphs with vertex sets V_1, V_2 respectively, then their cartesean product $G_1 \times G_2$ is the graph with vertex set $V_1 \times V_2$ such that (x_1, y_1) and (x_2, y_2) are adjacent in $G_1 \times G_2$ iff either $x_1 = x_2$ and y_1, y_2 are adjacent in G_2 or $y_1 = y_2$ and x_1, x_2 are adjacent in G_1. Show that $\chi(G_1 \times G_2) = \max(\chi(G_1), X(G_2))$. [20]

3. For $n \geq 2$, the Mobius ladder L_n is the graph with $2n$ vertices $x_1, ..., x_n, y_1, ..., y_n$ and $3n$ edges $\{x_i, y_i\}(1 \leq i \leq n), \{x_i, x_{i+1}\}, \{y_i, y_{i+1}\}(1 \leq i < n)$ and $\{x_n, y_1\}, \{y_n, x_1\}$. Show that L_n is not a matchstick graph. That is, it is not possible to draw it on the plane with vertices represented by $2n$ distinct points and edges represented by straight line segments of unit length. [20]

4. If ϑ, e, f are the number of vertices, edges and faces of a planar graph, then prove Euler's formula $\vartheta - e + f = 2$. Hence show that $e \leq 3(\vartheta - 2)$ when $\vartheta \geq 3$. Give an example of a six vertex planar graph attaining equality here. [20]

5. If G is a connected graph with diameter 2 and girth 5, then find the possible values of its degree. Show that there is a unique such graph of degree 3. [20]